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Optimal tracking of the water level for a coupled tank system using Linear Quadratic Regulator

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Abstract

It is proposed an optimal controller that will lead to zero asymptotic steady-state tracking error. The reference inputs can include steps, ramps, and other persistent signals. For a step signal, it is well known that zero steady-state tracking error can be achieved with a type-one open loop system. This idea is formalized in this work, by augmenting the coupled tank system model with an internal model of the reference input. Then, through the Linear Quadratic Regulator (LQR) method, the desired performance objectives are addressed by minimizing a quadratic function of the state and control input. Experimental results on the coupled tank system have been provided to illustrate the effectiveness of the method.

Keywords: Reference tracking, Optimal control, LQR, Internal model, Coupled tank system.

Seguimiento óptimo del nivel de agua de un sistema de tanques acoplados empleando el regulador lineal cuadrático

Resumen

Se propone un controlador óptimo que resultará en un error de seguimiento asintótico en estado estacionario nulo. Las entradas de referencia pueden incluir escalones, rampas y otras señales persistentes. Para una señal tipo escalón, es bien conocido que se puede lograr un error de seguimiento de estado estacionario cero con un sistema de lazo abierto de tipo uno. Esta idea se formaliza en este trabajo, aumentando el modelo del sistema de tanques acoplados con un modelo interno de la entrada de referencia. Luego, a través del regulador lineal cuadrático (LQR, por sus siglas en inglés), los requerimientos de diseño sobre la respuesta temporal son incorporados mediante la minimización de una función cuadrática dependiente de las variables de estado y de la señal de control. Se proporcionan resultados experimentales en el sistema de tanques acoplados para ilustrar la efectividad del método.

Palabras clave: Seguimiento de señales de referencia, Control óptimo, LQR, Modelo interno, Sistema de tanques acoplados.

Suivi optimal du niveau d'eau pour un système de réservoirs couplés utilisant le régulateur quadratique linéaire

Resumé

Il est proposé un contrôleur optimal qui produira une erreur de poursuite nulle en régime permanent asymptotique. Les entrées de consigne peuvent inclure des échelons, des rampes et d'autres signaux persistants. Pour un signal type échelon, il est bien connu qu'une erreur de poursuite en régime permanent nulle peut être obtenue avec un système à boucle ouverte de type un. Cette idée est formalisée dans ce travail, en augmentant le modèle du système de réservoirs couplés avec un modèle interne de l'entrée de référence. Ensuite, grâce au régulateur quadratique linéaire (LQR, pour son sigle en anglais), les objectifs de performance souhaités sont atteints en minimisant une fonction quadratique de l'état et de l'entrée de commande. Des résultats expérimentaux sur le système de réservoirs couplés ont été fournis pour illustrer l'efficacité de la méthode.

Mots clés: Suivi de référence, Commande optimale, LQR, Modèle interne, Système de réservoirs couplés.

Rastreamento ideal do nível de água para um sistema de tanque acoplado usando regulador quadrático linear

Resumo

É proposto um controlador ótimo que levará a zero erro de rastreamento assintótico em estado estacionário. As entradas de referência podem incluir degraus, rampas e outros sinais persistentes. Para um sinal de passo, é bem conhecido que o erro de rastreamento em estado estacionário zero pode ser alcançado com um sistema de malha aberta tipo um. Esta ideia é formalizada neste trabalho, aumentando o modelo do sistema de tanque acoplado com um modelo interno da entrada de referência. Em seguida, por meio do método do Regulador Quadrático Linear (LQR, por sua sigla em inglês), os objetivos de desempenho desejados são abordados minimizando uma função quadrática do estado e da entrada de controle. Resultados experimentais no sistema de tanque acoplado foram fornecidos para ilustrar a eficácia do método.

Palavras-chave: Rastreamento de referência, controle ideal, LQR, modelo interno, sistema de tanque acoplado

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i. INTRODUCCIÓN

Technologies such as chemical reactors, fermentation vessels, and steam and surge drums benefit from accurate level instrumentation and control [1, 2, 3, 4]. Tank level control systems are used frequently in different processes. For example: pharmaceutical industries, petrochemical plants, food/beverages industries and nuclear plants; depend upon tank level control systems. Often the tanks are coupled, so there is interaction between their levels, resulting in a nonlinear behavior and may also exhibit non minimum phase characteristics [5, 6].

There are numerous control policies developed for coupled tank systems. Among them, we can mention: Proportional-Integral-Derivative (PID) type controllers [7, 8, 9], Fuzzy control [10, 11], Model Predictive Control [12, 13], Backstepping Control [14, 15], Sliding-Mode Control [16, 17], Fractional PID type controllers [18, 19], Robust control [20] and Active Disturbance Rejection Control [21].

In this work, it is used Linear Quadratic Regulator Optimal Control (LQR) [22, 23]. This method is well known in modern optimal control theory and has been widely used in many applications [22]. LQR design has already been employed to control the liquid level of the tank system [24, 25, 26]. The relevance of this work aims to focus on determining an optimal solution by using the LQR method to the tank system tracking problem. Specially, it is desired to find asymptotic optimal tracking with zero steady-state error. This is formalized by introducing an internal model of the reference input [27, 28] in the state feedback control law previously obtained from a LQR problem.

The article is organized as follows. Section II describes the coupled tank system. In section III, the internal model principle and optimal LQR design are used to formulate a method that guarantees zero steady-state tracking error for the water level. Finally, in section IV, it is shown the effectiveness of the optimal design through its application to the coupled tank system and Conclusions.

ii. COUPLED TANK SYSTEM

The coupled tank system is given in Fig. 1. The setup experiment also includes a personal computer and the Matlab/Simulink software interface. The apparatus is used in the control laboratory at the Simón Bolívar University in Venezuela. It consists of a single pump with two tanks. Each tank is instrumented with a pressure sensor to measure the water level. The pump drives the water from the bottom basin up to the top of the system. Depending on how the outflow valves are configured, the water then flows to the top tank, bottom tank or both. One configuration is shown in Fig. 2, where the output of the pump is connected to the first tank.

The nonlinear state space model [6, 29] is shown in (1) where the state vector is equal to the tanks levels, the control signal corresponds to the input voltage applied to the pump and the output is chosen as the second tank level.

$$\dot{x}(t) = \begin{bmatrix} \dfrac{-C_1}{A_1}\sqrt{x_1(t)} & 0 \\ \dfrac{C_1}{A_2}\sqrt{x_1(t)} & \dfrac{-C_2}{A_2}\sqrt{x_2(t)} \end{bmatrix} x(t) + \begin{bmatrix} K_p/A_1 \\ 0 \end{bmatrix} u(t) \tag{1}$$

$$y(t) = [0 \quad 1]x(t)$$

With $C_1 = A_{d1}\sqrt{2g}$, $C_2 = A_{d2}\sqrt{2g}$, A_1 and A_2 denote the cross-sectional area of the tanks 1 and 2, respectively. A_{d1}, A_{d2} give the cross-sectional areas of the corresponding orifices, g is the gravitational constant on Earth and K_p is the pump flow constant.

The nonlinear model is linearize in the operating point $[\tilde{L}_1, \ \tilde{L}_2]^T$ resulting in the equations

$$\dot{x}(t) = \begin{bmatrix} \dfrac{-C_1}{2A_1\sqrt{\tilde{L}_1}} & 0 \\ \dfrac{C_1}{2A_2\sqrt{\tilde{L}_1}} & \dfrac{-C_2}{2A_2\sqrt{\tilde{L}_2}} \end{bmatrix} x(t) + \begin{bmatrix} K_p/A_1 \\ 0 \end{bmatrix} u(t) \tag{2}$$

$$y(t) = [0 \quad 1]x(t)$$

The description and numerical values of the physical parameters for the tank system are given in Table 1.

By employing the numerical values from Table 1. It is possible to perform the following calculations

$$A_1 = A_2 = \pi(4.445/2)^2 = 15.53 \ cm^2$$
$$A_{d1} = \pi(0.635/2)^2 = 0.317 \ cm^2$$
$$A_{d2} = \pi(0.476/2)^2 = 0.178 \ cm^2$$
$$\tilde{L}_1 = \tilde{L}_2 = 15 \ cm$$

Replacing the previous values in (2) allows to obtain the linear model of the coupled tank system as

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$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{3}$$

where

$$A = \begin{bmatrix} -a & 0 \\ a & -b \end{bmatrix}, B = \begin{bmatrix} c \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

With $a = 0.1168, b = 0.0656$ and $c = 0.2577$.

Table I. Physical parameters of the coupled tank system.

Description	Value	Unit
Pump flow constant	4	$cm^3/s/V$
Out 1 Orifice Diameter	0.635	cm
Out 2 Orifice Diameter	0.476	cm
Tanks Diameter	4.445	cm
Tanks Height	30	cm
Gravitational constant on Earth	981	cm/s^2
Maximum flow	100	cm^3/s
Pump peak voltage	22	V

Figure 1. Coupled tank system.
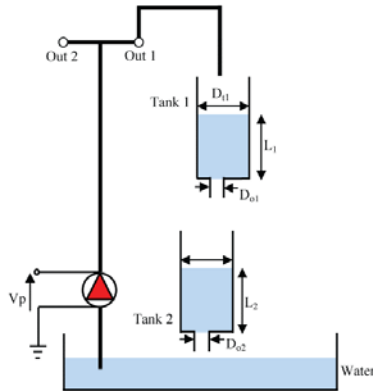
Figure 2. Standard configuration of the coupled tank system.

iii. OPTIMAL TRACKING CONTROL SYSTEM DESIGN

We begin by considering the design problem to enable the optimal tracking of a step reference input $y_r(t) = r$ with zero steady-state error $e(t)$ defined as

$$e(t) = y_r(t) - y(t) \tag{4}$$

Taking the time-derivative of the auxiliary signal $\varepsilon(t) = -e(t)$ and using the output equation in (3) yields

$$\dot{\varepsilon}(t) = \dot{y}(t) - \dot{r} = C\dot{x}(t) - 0 = C\dot{x}(t)$$

Applying now, time-derivative to the state equation in (3) and defining the intermediate variables $v(t) = \dot{x}(t)$ and $u_0(t) = \dot{u}(t)$ leads to the step tracking system

$$\begin{bmatrix} \dot{\varepsilon}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \begin{bmatrix} \varepsilon(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u_0(t) \tag{5}$$

Or equivalently, in compact form

$$\dot{z}(t) = A_a z(t) + B_a u_0(t) \tag{6}$$

for the vector $z(t) = [\varepsilon(t) \quad v(t)]^T$ and augmented matrices

$$A_a = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, B_a = \begin{bmatrix} 0 \\ B \end{bmatrix}$$

In order to have a LQR formulation of the tracking problem, the following quadratic cost is considered

$$J = \int_0^\infty [z(t)^T Q z(t) + \rho u_0(t)^2] \, dt \tag{7}$$

where Q is a positive semi-definite matrix which has an impact on the closed-loop transient response and parameter $\rho > 0$ can be used to tune the amplitude of the control signal. It is well known [22, 23] that the state feedback control

$$u_0(t) = -Kz(t) \tag{8}$$

minimizes Eq. (7) and stabilizes system (6) with the vector K equal to

$$K = -\rho^{-1} B_a{}^T P \tag{9}$$

Here P is the symmetric positive definite solution of the Continous Algebraic Riccati Equation given by

$$A_a{}^T P + P A_a - \rho^{-1} P B_a B_a{}^T P + Q = 0 \tag{10}$$

Since, it can be easily verified that the system (5) with matrices A, B and C from (3) is controllable [30]. We can find constants K_1 and K_2 by expressing gain vector $K \in \mathbb{R}^{1x(n+1)}$ as $K = \begin{bmatrix} K_1 \\ 1x1 & K_2 \\ 1xn \end{bmatrix}$ in (8). The resulting equation is

$$u_0(t) = -[K_1 \quad K_2] \begin{bmatrix} \varepsilon(t) \\ v(t) \end{bmatrix} \tag{11}$$

After performing time-integration in (11), the control signal $u(t)$ in (3) applied to the coupled tank system is

$$u(t) = K_1 \int_0^t e(\tau)\, d\tau - K_2 x(t) \qquad (12)$$

Equation (12) guarantees system (6) is stable and the quadratic cost (7) is minimized. The above results are summarized in Theorem 1 and the corresponding control system block diagram in Fig. 3

Theorem 1: If the state feedback control law (12) is applied to the coupled tank system. The error signal (4) for a step reference input $y_r(t) = r$ will tend asymptotically to zero minimizing the quadratic cost (7) for a given positive semi-definite matrix Q and parameter $\rho > 0$.
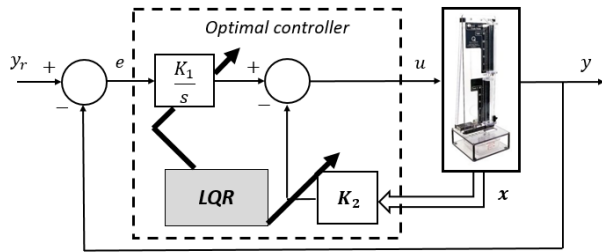

Figure 3. Optimal LQR internal model design for the coupled tank system for a step reference input.

It is straightforward to extend the method for a ramp reference input $y_r(t) = t,\ t \geq 0$. Taking twice the time-derivative of $\varepsilon(t) = -e(t)$ and using (3) yields

$$\ddot{\varepsilon}(t) = \ddot{y}(t) - \ddot{y}_r(t) = C\ddot{x}(t) - 0 = C\ddot{x}(t)$$

Defining the intermediate variables $v(t) = \ddot{x}(t)$ and $u_0(t) = \ddot{u}(t)$ gives

$$\begin{bmatrix} \dot{\varepsilon}(t) \\ \ddot{\varepsilon}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & C \\ 0 & 0 & A \end{bmatrix} \begin{bmatrix} \varepsilon(t) \\ \dot{\varepsilon}(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} u_0(t) \qquad (13)$$

Again, the previous equation can be expressed in the compact form given by (6) if the following augmented matrices are defined

$$A_a = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & C \\ 0 & 0 & A \end{bmatrix}, B_a = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$$

Since, system (13) with matrices A, B and C from (3) is controllable [30]. We can compute constants K_1, K_2 and K_3 in (8) to form

$$u_0(t) = -K_1 \varepsilon(t) - K_2 \dot{\varepsilon}(t) - K_3 \ddot{x}(t) \qquad (14)$$

The control signal $u(t)$ applied to the coupled tanks system is found by integrating (14) twice, this signal will cause the error signal (4), for a ramp reference input, tend asymptotically to zero minimizing the

quadratic cost (7) for a given positive semi-definite matrix Q and parameter $\rho > 0$. Fig. 4 shows the block diagram of the control system.
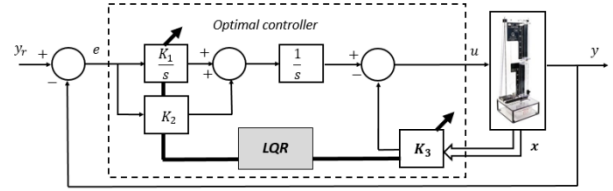

Figure 4. Optimal LQR internal model design for the coupled tank system for a ramp reference input.

iv. DESIGN TESTS ON A LABORATORY SETUP

The design technique is demonstrated through the implementation on the coupled tank system in real time and the corresponding results are presented in this section.

A. Step reference input

The reference input applied to the coupled tank system is shown in Fig. (5). It consists of two changes in the set point, the first at $60\ s$ and the second at $160\ s$, after the start of the experiment.
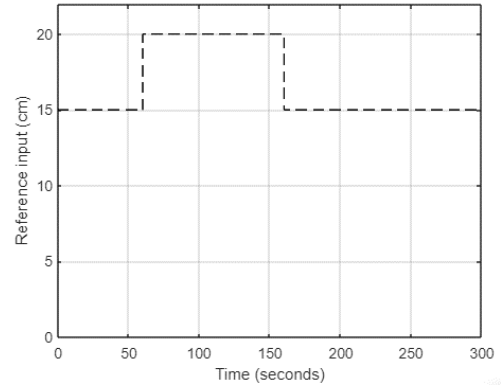

Figure 5. Input reference signal to validate the optimal controller design with step-like internal model.

Table 2 collects several experiments carried out to study the impact of the selection of matrix Q and the constant ρ in (7) on the closed-loop performance of the system. The performance measures considered are: rise time (t_r), maximum percentage overshoot $(\%M_p)$, integral square error (ISE) and root mean square of the control signal (u_{RMS}) [31].

Gain K in (8) is computed using the *lqr* function of Matlab. The syntax of the command is $K = lqr(A_a, B_a, Q, \rho)$. Experiments 1 to 4 show the influence of the constant ρ in (7) on the amplitude

of the control signal applied to the coupled tank system. This is confirmed by the decrease in the value of u_{RMS} as ρ increases. Although the energy of the control effort decreases, on the other hand, the transient response tends to deteriorate slightly. The remaining experiments evaluate the contribution of the matrix Q in the transient response of the liquid level of the tank. The increase in the values of the diagonal of the matrix generates a faster system response at the expense of a greater overshoot. The time responses of the seven experiments of Table 2, from a practical point of view, are satisfactory. In order not to overload the article with an excessive amount of graphics, we select the first experiment to display some results. Figure 6 confirms the good tracking of the liquid level in the tank and Fig. 7 shows the control signal applied to the pump of the tank always within the voltage technological limits of the pump.

B. Ramp reference input

The reference input applied to the coupled tank system is shown in Fig. 8.The seven experiments carried out to evaluate the ramp internal model and LQR design are summarized in Table 3. Each controller designed exhibits excellent results. In this case, we choose the response of the seventh experiment to display the performance of the control system, through figures 9 and 10.

Table II. Different experiments implemented on the coupled tank system to validate the optimal design of the controller for a step-type internal model in the reference input.

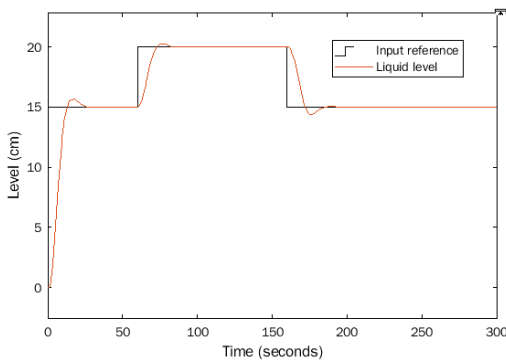| #	Q, ρ	t_r	$\%M_p$	ISE	u_{RMS}
1	$Q = I, \rho = 0.5$	23.70	**4.53**	1411	6.59
2	$Q = I, \rho = 1$	28.44	5.13	1541	5.64
3	$Q = I, \rho = 2$	30.81	5.53	1709	5.54
4	$Q = I, \rho = 4$	38.71	5.80	1912	**5.50**
5	$Q = 5I, \rho = 1$	17.38	8.13	1308	6.59
6	$Q = 10I, \rho = 1$	11.85	14.00	**1291**	6.62
7	$Q = 20I, \rho = 1$	**11.06**	21.53	1354	6.76

Figure 6. Closed-loop liquid level response for experiment 1 of Table: 2.
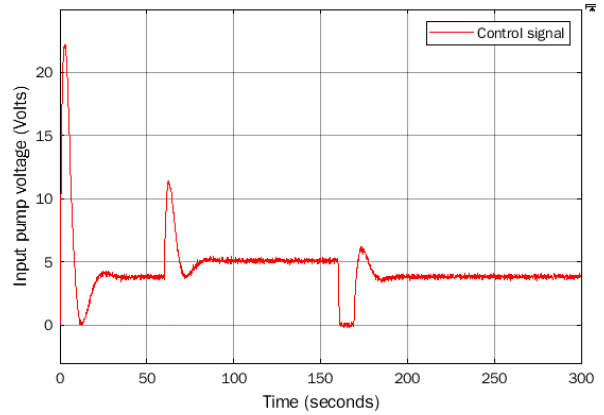

Figure 7. Control signal for experiment 1 of Table 2.

C. Disturbance rejection

The disturbance rejection ability of the design from the experiment 1 of the step reference case is now studied through the following experiment. The system starts in the configuration of Fig. 2, but at $t = 100\ s$ switches to that of Fig. 11, where the pump output feeds both tank 1 and tank 2 and at $t = 200\ s$, it returns to the initial interconnection. This experiment allows to model a trapezoidal perturbation that operates in the interval $[100; 200]\ s$, causing a decrease in the inlet flow to tank 1 and the appearance of a direct flow in tank 2. Figure 12 shows the very satisfactory property of disturbance rejection achieved with the design method.
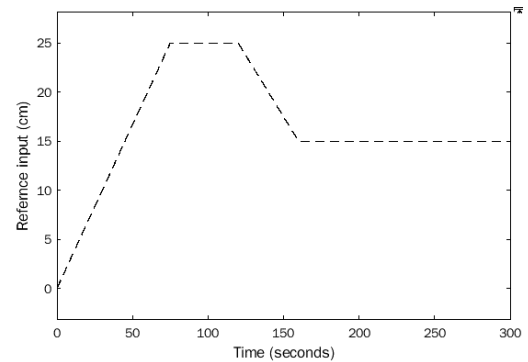

Figure 8. Input reference signal to validate the optimal controller design with ramp-like internal model.

Table III. Different experiments implemented on the coupled tank system to validate the optimal design of the controller for a ramp-type internal model in the reference input.

#	Q, ρ	$\%M_p$	ISE	u_{RMS}
1	$Q = I, \rho = 0.5$	4.00	15.5	4.74
2	$Q = I, \rho = 1$	4.32	19.7	4.74
3	$Q = I, \rho = 2$	4.68	25.1	4.73
4	$Q = I, \rho = 4$	5.10	32.6	**4.72**
5	$Q = 5I, \rho = 1$	3.60	11.5	4.74

6	$Q = 10I, \rho = 1$	3.32	9.5	4.74
7	$Q = 20I, \rho = 1$	**3.08**	**7.8**	4.74
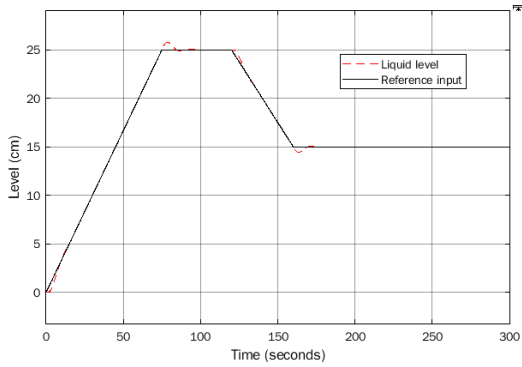
Figure 9. Closed-loop liquid level response for experiment 7 of Table 3.

Figure 11. Connection scheme of the coupled tanks system to create a disturbance that decrease the inlet to tank 1.

v. CONCLUSIONS

It is proposed a method to design a controller that guarantees the optimal tracking of the water level in a coupled tank system. The major advantage of the proposed design is the framework on which it is based, the combination of the internal model principle and the LQR optimal method, provides simplicity and flexibility in tuning the small number of controller gain parameters that are necessary for the implementation of the method. The experiments are performed on the coupled tank system and the results illustrate the effectiveness of the method.

Currently, research is being carried out on how to transfer specifications of the desired closed-loop transient response, for example, over shoot and settling time, in elements of the **Q** matrix of the quadratic cost of the LQR technique.
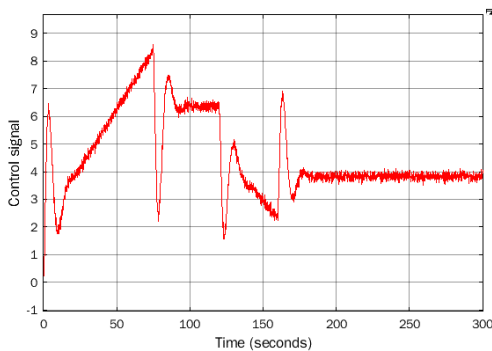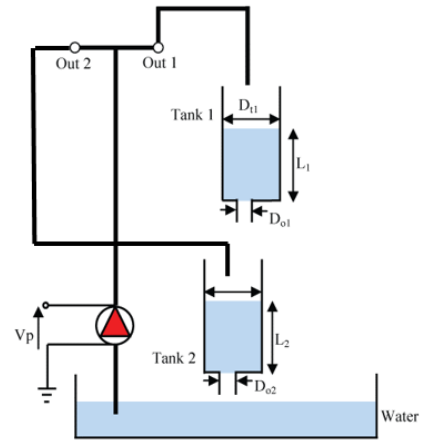

Figure 12. Closed-loop liquid level response for disturbance rejection.

ACKNOWLEDGMENTS

Figure 10. Control signal for experiment 7 of Table 3.

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